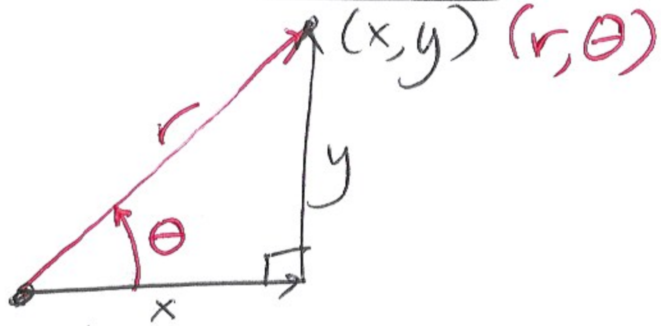
10.7 POLAR COORDINATES



ORIGIN/POLE

$(r,\theta) \longrightarrow (x,y)$

$x = r\cos\theta$

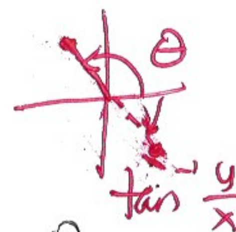
$y = r\sin\theta$

$(x,y) \longrightarrow (r,\theta)$

$r^2 = x^2 + y^2 \longrightarrow r = \sqrt{x^2+y^2}$

$$\tan\theta = \frac{y}{x} \longrightarrow \theta = \begin{cases} \tan^{-1}\frac{y}{x}, & \text{IF } x>0 \\ \tan^{-1}\frac{y}{x} + \pi, & \text{IF } x<0 \\ \frac{\pi}{2}, & \text{IF } x=0 \text{ AND } y>0 \\ \frac{3\pi}{2}, & \text{IF } x=0 \text{ AND } y<0 \end{cases}$$

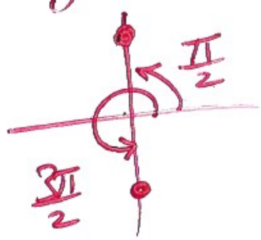
~~, IF $x=0$ AND $y=0$~~



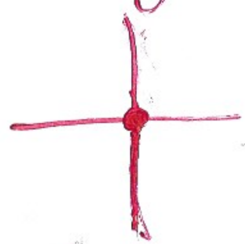
RANGE OF $\tan^{-1}x$ IS $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

IE. Q_1 OR Q_4

$x=0 \rightarrow$ y-AXIS



$x=0, y=0$



CONVERT $(x,y) = (-2\sqrt{3}, -6)$ TO POLAR

$$r = \sqrt{(-2\sqrt{3})^2 + (-6)^2}$$
$$= \sqrt{12 + 36}$$
$$= \sqrt{48}$$
$$= 4\sqrt{3}$$

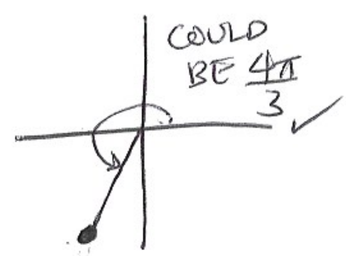
$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$$\sqrt{48} = 2 \cdot 2\sqrt{3}$$
$$= 4\sqrt{3}$$

$$\theta = \pi + \tan^{-1}\frac{-6}{-2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \pi + \tan^{-1}\frac{3\sqrt{3}}{3}$$
$$= \pi + \tan^{-1}\sqrt{3}$$
$$= \pi + \frac{\pi}{3}$$
$$= \frac{4\pi}{3}$$

$$(r,\theta) = \left(4\sqrt{3}, \frac{4\pi}{3}\right)$$

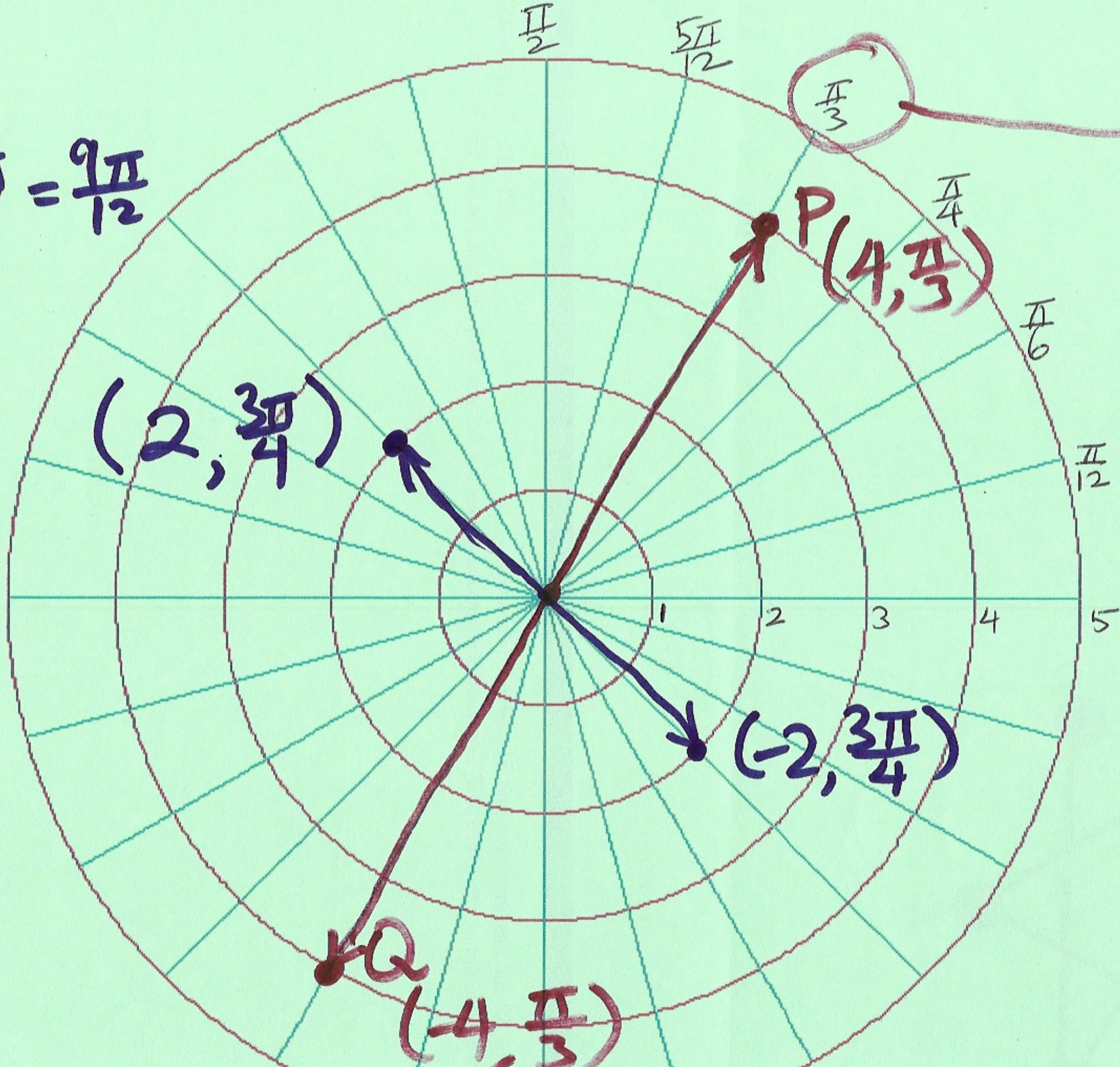
SANITY CHECK:



$\frac{4\pi}{3} \in Q_3$ ✓

$\sqrt{3} \approx 1.8$
$2\sqrt{3} \approx 3.6$
$(-2\sqrt{3}, -6)$
$\approx (-3.6, -6)$
LESS LEFT MORE DOWN

$\frac{3\pi}{4} = \frac{9\pi}{12}$



OR

$\left(4, \frac{\pi}{3} + \pi\right) = \left(4, \frac{4\pi}{3}\right)$

CONSIDER POLAR POINT

$P\left(4, \frac{\pi}{3}\right)$

NOW CONSIDER

$Q\left(-4, \frac{\pi}{3}\right)$

CONVERT P TO RECTANGULAR

$x = 4\cos\frac{\pi}{3} = 4 \cdot \frac{1}{2} = 2$

$y = 4\sin\frac{\pi}{3} = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$

$(x, y) = (2, 2\sqrt{3})$

CONVERT Q TO RECTANGULAR

$x = -4\cos\frac{\pi}{3} = -2$

$y = -4\sin\frac{\pi}{3} = -2\sqrt{3}$

$(x, y) = (-2, -2\sqrt{3})$

POLAR POINT $(-r,\theta)$ IS DIAMETRICALLY OPPOSITE THE POINT (r,θ)

↑ REFLECTED OVER POLE

$(-r,\theta)$ IS THE SAME POINT AS $(r,\theta+\pi)$

(r,θ) IS THE SAME POINT AS $(-r,\theta+\pi)$

(r,θ) IS THE SAME POINT ~~AND~~ AS $(-r,\theta+\pi+2n\pi)$
AND $(r,\theta+2n\pi)$
FOR ALL $n\in\mathbb{Z}$

GIVEN THE POINT $(r,\theta)=(7,\frac{\pi}{5})$
FIND POLAR COORDINATES FOR THIS POINT WITH

1. $r>0,\theta<0$ $\quad (r,\theta)=(7,\frac{\pi}{5}-2\pi)=(7,-\frac{9\pi}{5})$
2. $r<0,\theta>0$ $\quad (r,\theta)=(-7,\frac{\pi}{5}+\pi)=(-7,\frac{6\pi}{5})$
3. $r<0,\theta<0$ $\quad (r,\theta)=(-7,\frac{6\pi}{5}-2\pi)=(-7,-\frac{4\pi}{5})$

CONVERT RECTANGULAR EQUATION TO POLAR

SUBSTITUTE $x = r\cos\theta$

$y = r\sin\theta$

eg. CONVERT $y = x^2$ TO POLAR

$$r\sin\theta = (r\cos\theta)^2$$

$$r\sin\theta = r^2\cos^2\theta$$

$$\frac{\sin\theta}{\cos^2\theta} = \frac{r^2}{r}$$

$$r = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$$

$$r = \tan\theta \sec\theta$$

$$\boxed{r = \sec\theta \tan\theta}$$

eg. CONVERT $y = 2x + 1$ TO POLAR

$$r\sin\theta = 2r\cos\theta + 1$$

$$r\sin\theta - 2r\cos\theta = 1$$

$$r(\sin\theta - 2\cos\theta) = 1$$

$$\boxed{r = \frac{1}{\sin\theta - 2\cos\theta}} \neq \frac{1}{\sin\theta} - \frac{1}{2\cos\theta}$$

$$\frac{1}{3-2} \neq \frac{1}{3} - \frac{1}{2}$$

$$1 \neq -\frac{1}{6}$$